COURSE DETAIL

STELLAR DYNAMICS

Country
Italy

Host Institution
University of Bologna

Program(s)
University of Bologna

UCEAP Course Level
Upper Division

UCEAP Subject Area(s)
Physics

UCEAP Course Number
169

UCEAP Course Suffix

UCEAP Official Title
STELLAR DYNAMICS

UCEAP Transcript Title
STELLAR DYNAMICS

UCEAP Quarter Units
6.00

UCEAP Semester Units
4.00

Course Description

This course is part of the Laurea Magistrale degree program and is intended for advanced level students. Enrollment is by permission of the instructor. This course covers the theoretical understanding and working knowledge of the principal gravitational phenomena determining the structure, the dynamics and the evolution of stellar systems, from open and globular clusters, to galaxies, to galaxy clusters. At the end of the course, the student is able to use in autonomy some of the advanced mathematical techniques needed in potential theory and in epicyclic theory. The course content is divided into 2 parts:

1. **GENERALS**

Gravitational field of point particles, principle of superposition. Integral representation for any distributions. Most important properties of the divergence operator and its coordinate-free representation starting from Gauss's Theorem. Operational introduction to the one-dimensional and multidimensional Dirac Delta in Cartesian and curvilinear coordinates. Calculation of the divergence of the field of extended distributions, Poisson's equation for the field. Direct proof of the First and Second Newton's theorem (homogeneous spherical shells). Alternative demonstration using Gauss's theorem. Coordinate-free representation of the gradient, curl, and Laplacian operators. Notes on differential forms. Exact fields and their properties, potential and work. Closed fields. Stokes' theorem, closed fields in simply and non-simply connected domains. Existence of the potential and its connection with the total energy of a particle. Potential difference as a line integral. Formal calculation of the potential of a point mass. Potential of extended distributions, general expression and discussion of the meaning of the additive constant. Poisson and Laplace equations. First and second Green's identities, uniqueness of the solution of the Poisson equation in bounded volumes with prescribed boundary conditions. Field inside cavities with equipotential boundary. Helmholtz Decomposition Theorem. Definition of concentric and similar ellipsoids. Definition of homoeoid. Statement of the Third Newton's Theorem for finite homoeoids. Field inside a heterogeneous hollow homoeoid from the principle of superposition. Co-area theorem,

relationship with the field of homoeoids. Definition of confocal ellipsoidal coordinates. Classification of the three families of associated quadrics. Ellipsoidal coordinates: orthogonality, gradient, Laplacian. Application to the problem of the ellipsoidal layer with zero internal field. Potential of the heterogeneous ellipsoid. Chandrasekhar's formula. Introduction to the multipole expansion of potential in the far field. Monopole, dipole and quadrupole terms.

Introduction to the concept of Green's function for linear differential operators and their use in solving nonhomogeneous problems. The potential of a material point as an explicit example of a Green's function for the Laplacian. Separation of variables for the Laplacian in Cartesian coordinates. Fourier transform and inverse transform in Rn, the case of the Dirac Delta. Green's function in Cartesian coordinates. Green's function in spherical coordinates. Separation of variables. Rotational invariance and the azimuthal quantum number m. Orthogonality of azimuthal functions. Associated Legendre equation for the latitude angle, transformation into an algebraic equation. Outline with examples of singularities of ODEs, both mobile and fixed. Fuchs' theorem, regular points, regular singularities, and essential singularities. Classification for the Legendre equation. Frobenius method and polar quantum number. Legendre functions and associated functions P and Q. Legendre polynomials. Rodrigues formulas, norm of associated polynomials. Orthogonality of solutions with Sturm-Liouville theory. Spherical harmonics as eigenfunctions of the angular part of the Laplacian. Systems with cylindrical symmetry. Generating function for Legendre polynomials, multipole moments. Gegenbauer polynomials. Addition theorem for spherical harmonics. Separation of variables for the vacuum solution of the Laplacian in cylindrical coordinates. Bessel equation and its properties: orthogonality of solutions, singular points. Asymptotic analysis of Bessel functions for large values of the argument. Closure relation and Hankel transform. Green's function in cylindrical coordinates for the Laplacian. Any density potential with Fourier-Bessel transforms. Case of axisymmetric systems. Infinitely thin axisymmetric disks, potential in the plane of the disk, homogeneous rings. Thin disk rotation curve. Mestel's disc and exponential, implications for the dark matter halos. Potential of

axisymmetric systems using elliptical integrals.

2. COLLISIONLESS SYSTEMS

Introduction to the epicyclic approximation. Notes on curvilinear coordinates, velocity and acceleration in cylindrical coordinates. Newtonian equations of motion in general axisymmetric potentials, conservation of energy and Jz. Deduction of equations from the Euler-Lagrange equations. The meridional plane, its motion, and effective potential. Equations of motion in the meridional plane, orbital families, circular orbits and their (equivalent) equations. Interpretation of total energy as energy for motion in the meridional plane, extremum properties for the energy of circular orbits, centrifugal barrier, zero-velocity curves. Development of the effective potential to second order. Frequency of vertical and radial epicycles. Radial and vertical motion on the epicycle in the case of stable orbits, zero-velocity ellipses. Rayleigh criterion and examples of applications. First-order angular motion, coordinates on the equatorial plane referred to the deferent, equation of the epicycle on the equatorial plane, and determination of the axes for the epicyclic ellipse. Epicycles in Coulomb, harmonic, and flat rotation potentials: frequency and shape. Relation of Oort constants to the radial epicyclic frequency. Closed, rosette, and open orbits: closure conditions, pattern angular velocity, Lindblad kinetic waves, and the dynamical phenomenology of disks.

Language(s) of Instruction
English

Host Institution Course Number
96386

Host Institution Course Title
STELLAR DYNAMICS

Host Institution Course Details
https://www.unibo.it/en/study/course-units-transferable-skills-moocs/course-uni...

Host Institution Campus

BOLOGNA

Host Institution Faculty

Host Institution Degree
LM in ASTROPHYSICS AND COSMOLOGY

Host Institution Department
PHYSICS AND ASTRONOMY

Course Last Reviewed
2025-2026

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